Apparent Mass of Small Children : Modelling

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Abstract

Mass-spring-damper models are widely available for quantifying the whole-body vibration characteristics of primates, human adolescents and human adults, but no models have previously been developed for small children. In this study a single degree of freedom, linear, mass-spring-damper with base support model was determined from the seated vertical apparent mass modulus function of each of eight small children of less than 18 kg in mass. A Differential Evolution optimisation algorithm was used in conjunction with a mean squared error measure and penalty functions to identify the optimal child model parameter values. The eight child models were characterised by a mean moving mass m_1 of 8.5 kg, a mean body stiffness k_1 of 21131 N/m and a mean damping coefficient c_1 of 329 Ns/m. Comparison to the parameter values of similar models reported in the literature for Rhesus monkeys, Baboons, large children and adults suggests that the values obtained in the current study for small children are intermediate between the smaller primates and the larger humans. A regression analysis of the model parameters was performed as a function of subject mass for a data set consisting of the eight child models, twelve similar models for primates, and 60 similar models for large children and adults. The moving mass m_1 of the group of models grew with a power exponent of approximately unity, the body stiffness k_1 grew with a power exponent of approximately +1/2, the damping coefficient c_1 grew with a power exponent of approximately +3/4 and the dimensionless damping ratio was independent of subject mass. The natural frequency of the models grew with a power exponent of approximately –1/4.

Keywords: children, vibration, model, whole-body, seat

1. Introduction

Economical use of human vibration response data requires a synthetic representation in the form of an analytical model. These have most often consisted of mass-spring-damper models involving a finite number of elements. Such models can serve one of two basic purposes: (1) to represent the dynamic loading of the body to a sufficient degree to permit its simulation during the testing of objects such as seats and (2) to represent the inner structures of the body to a sufficient extent to aid the understanding of human motion and of the potentially damaging effects of vibration. Models used for the first purpose can be described as impedance or loading models, which represent the effect of the body on engineering structures. Models developed for the second purpose can be described as human response models, representing to some degree of accuracy the biological components which determine the whole-body response.

Since the 1950s numerous mass-spring-damper representations of the adult human body have been developed and used for applications of both the first and the second type. Important investigations which have defined whole-body mass-spring-damper models for either vibration or shock include the studies by Lantham (1957), Coermann et. al. (1960), Coermann (1961), Payne (1961), Coerman (1962), Von Gierke (1964), Toth (1966), Wittmann and Phillips (1969), Suggs et. al. (1969), Hopkins (1970), Payne and Band (1971), Potemkin and Frolov (1972), Vogt et. al. (1973 and 1978), Muksian and Nash (1974 and 1976), Garg and Ross (1976), Payne (1978), Mertens (1978), Nigam and Malik (1987), Amirouche (1987), Fairley and Griffin (1989), Smith (1994), Knoblauch et. al. (1995), Pankoke et. al. (1998), Wei and Griffin (1998), Boileau and Rakheja (1998), Mansfield and Lunström (1999), Matsumoto and Griffin (2001) and Boileau et. al. (2002). The majority of the reported mass-spring-damper models have consisted of linear systems, important exceptions being the nonlinear models described by Wittman and Phillips (1969), Hopkins (1970), Payne and Band (1971), Muksian and Nash (1976) and Mertens (1978). Most mass-spring-damper models have involved few degrees of freedom, typically from one to four, important exceptions being the models defined by Coermann et. al. (1960), Von Gierke (1964), Toth (1966), Muksian and Nash (1974), Garg and Ross (1976), Mertens (1978), Nigam and Malik (1987), Amirouche (1987), Smith (1994), Knoblauch et. al. (1995) and Pankoke et. al. (1998). The vibration exposure most often considered has been seated or standing vibration in the vertical direction, important exceptions being the supine models defined by Vogt et. al. (1973 and 1978) and the seated horizontal direction models defined by Mansfield and Lundström (1999).

The need for summarising the large number of available models has lead the International Standards Organisation to specify mass-spring-damper representations. 1981 saw the publication of *International Standards Organisation 5982 (1981): Vibration and shock – Mechanical driving point impedance of the human body*. Alongside tabulated vertical direction impedance values for the seated, standing and supine postures, the standard also specified mass-spring-damper models for use in the frequency range from 0.5 to 31.5 Hz. A separate model was provided for each posture. Of note was the use of a three degree of freedom model for the supine posture as opposed to two degree of freedom representations for sitting and standing. 1987 saw the publication of *ISO Standard 7962 (1987): Vibration and shock – Mechanical transmissibility of the human body* which was intended as a summary of the then existing literature on seat-to-head and floor-to-head transmissibility. Besides tabulated modulus and phase data, the standard also provided a four degree of freedom mass-spring-damper model of the body. The movement of mass m_1 of the model was suggested for use in calculating seat-to-head and floor-to-head transmissibility. ISO standard 7962 remained in publication until amalgamated into the 2001 revision of ISO 5982 *"Mechanical vibration and shock - range of idealised values to characterise seated-body biodynamic response under vertical vibration"*. The new edition defined minimum, maximum and mean curves of driving point mechanical impedance, apparent mass and seat-to-head transmissibility A three degree of freedom mass-spring-damper model was provided to represent the body in the vertical direction. The model was stated to be representative of (a) a posture described as erect seated without backrest support, with feet supported and vibrated, (b) subject mass in the range from 49 to 93 kg and (c) unweighted sinusoidal or random input acceleration amplitudes between 0.5 and 3.0 m/s^2 with the predominance of frequencies within the range from 0.5 to 20 Hz. The movement of mass m_2 of the model was suggested for use in determining seat-to-head transmissibility.

A characteristic of all previously reported models is that the human subjects consist of either adults or adolescents. The youngest subjects previously reported were a group of twelve children with ages in the range from 7 to 14 years, who were tested by Fairley and Griffin (1989) and modelled as both single and dual degree of freedom mass-spring-damper systems by Wei and Griffin (1998). These children were outside the age group associated with the use of products such as child safety seats. In addition, their physical size would suggest that their vibration response properties might be similar to those of adults. A research question that has remained unanswered is what might be the whole-body vibration characteristics of small children, under 18 kg in mass, who use products such as child safety seats.

In the case of the small children some insights can be gained from studies involving similarly sized primates. For Rhesus monkeys (macaca mulatta) an investigation by Broderson and Von Gierke (1971) produced a two-mass, single degree of freedom, linear, mass-spring-damper model of a seated 14.5 pound subject. Comparison of the primate model to the models reported for seated adult humans suggests differences in terms of both the frequency of resonant response and the number of degrees of freedom. Whereas the first resonance frequency of seated adult humans has normally been reported to be in the interval from 4 to 6 Hz, that of the Rhesus primate tested by Broderson and Von Gierke was 8.4 Hz. In addition, while the most commonly encountered mass-spring-damper model for adult humans is a two degree of freedom system, the Broderson and Von Gierke primate model had a single degree of freedom. Edwards et. al. (1976) presented vertical impedance data from tests of four Rhesus monkeys and eighteen dogs in sitting postures. Again, the test data suggested that the first resonance frequency for all four primates was in the interval from 7 to 8 Hz and that the response was characterised by a single degree of freedom. Two investigations published by Slonim (1985 and 1987) presented seated vertical impedance curves and both seat-to-L5 and seat-to-T3 transmissibility for four Rhesus monkeys and three baboons. In all cases the data suggested a whole-body response resonance in the interval from 6 to 12 Hz for the Rhesus monkeys and 5 to 10 Hz for the Baboons. A single degree of freedom mass-spring-damper model was found appropriate for the Rhesus primates, while the Baboons were found to be better represented by a dual degree of freedom system. A study by Smith (1992) measured the driving point mechanical impedance of four seated Rhesus monkeys. A single degree of freedom mass-spring-damper model was found to provide an accurate fit to the data of each subject, providing resonant frequencies in the interval from 9 to 11 Hz. A study by Smith and Kazarian (1994) performed similar measurements for three seated Rhesus primates and on this occasion identified two impedance peaks for each subject, the first at approximately 5 Hz and the second in the interval from 9 to 14 Hz. A three mass, dual degree of freedom, mass-spring-damper model was found to accurately represent the Rhesus primates on this occasion.

Review of the data for small primates and for adult humans suggests differences between the two groups in terms of both the nature (single degree of freedom rather than dual degree of freedom) and the frequency (first resonance in interval 6-14 Hz rather than 4-6 Hz) of resonant response. In addition, available models for the two groups have different mass, stiffness and damping values. In consideration of the uncertainty regarding the possible model parameters, and in light of the possible importance of this knowledge towards the correct design of products such as child safety seats, it was decided to analytically model an experimentally measured data set first described by Giacomin (2004). This paper presents the parameter identification procedure adopted and the models obtained.

2. Mass-Spring-Damper Model of Small Children

For a vibrating mechanical system the driving point apparent mass function is defined as

$$Apparent\,Mass\,(j\omega) \;=\; AM\,(j\omega) \;=\; \frac{F\,(j\omega)}{\ddot{x}\,(j\omega)} \qquad (1)$$

where $F\,(j\omega)$ and $\ddot{x}\,(j\omega)$ are the Fourier transforms of the force and acceleration measured at the point of input to the system under investigation. As described by Giacomin (2004), vertical direction seated apparent mass functions have been estimated for each of eight small children of mass less than 18 kg whose general characteristics are presented in Table 1. For each test subject, force and acceleration time histories were measured at the interface between a rigid seat and a vibrating platform using band-limited Gaussian random vibration defined over the frequency interval from 1 to 50 Hz and defined by two test amplitudes: 0.8 and 1.2 m/s^2. An H_v spectral estimate of the apparent mass function was determined using the acceleration and force time histories, which were sampled at 200 Hz. A 512 point Hanning window was used with an overlap of 97%, providing a spectral resolution of 0.39 Hz.

The experimentally measured vertical direction apparent mass modulus and phase functions are presented as Figures 1 and 2. These suggest a single degree of freedom whole-body response in the case of all children except subject “sa”, the oldest and largest of the group, whose response provided some evidence of a possible small contributing second degree of freedom. Across the eight data sets the frequency of peak apparent mass modulus occurred in the interval from 5.86 to 7.42 Hz, with a mean value of 6.25 Hz. The peak modulus varied from 8.40 to 20.2 kg, with a mean value of 14.8 kg. Since a single degree of freedom model appeared appropriate for use with the experimental data set considered, and since a comparison was sought with the widest possible range of existing models found in the literature, a single degree of freedom with base support model was adopted for the purposes of the current study. Comparison of the child apparent mass functions measured at the two input excitation levels of 0.8 and 1.2 m/s^2 suggested a degree of linearity, thus a linear mass-spring-damper model was chosen. The model chosen, shown in Figure 3, was therefore of the type widely applied to primates since the studies of Broderson and Von Gierke, and also occasionally used to represent adult humans since the studies by Fairley and Griffin.

[Insert Table 1 Here]

[Insert Figure 1 Here] **[Insert Figure 2 Here]** **[Insert Figure 3 Here]**

For the single degree of freedom system with base support the expression for the driving point apparent mass can be shown to be

$$AM(j\omega) = (m_o + m_1)\left(\frac{k_1 + c_1\omega j}{k_1 - m_1\omega^2 + c_1\omega j}\right) \tag{2}$$

which has modulus

$$|AM(j\omega)| = \sqrt{\frac{((m_o + m_1)k_1 - m_o m_1 \omega^2)^2 + ((m_o + m_1)c_1\omega)^2}{(k_1 - m_1\omega^2)^2 + (c_1\omega)^2}} \tag{3}$$

and phase

$$\angle AM(j\omega) = \tan^{-1}\left(\frac{(m_o + m_1)c_1\omega}{(m_o + m_1)k_1 - m_o m_1\omega^2}\right) - \tan^{-1}\left(\frac{c_1\omega}{k_1 - m_1\omega^2}\right) \tag{4}$$

3. Parameter Identification Procedure

The model defined by equation 2 was implemented in the MATLAB® software and its parameter values were identified for each child by minimising the difference between the apparent mass modulus of the model and that of the experimental data. Phase information was not considered due to the lower accuracy of the experimental estimates with respect to the modulus estimates. The differences between the model predicted data and the experimental data were quantified by means of the non-normalised mean squared error (m.s.e.)

$$m.s.e \;=\; \sum_{i=1}^{N}\left(\left|AM(j\omega)\right|-\left|A\hat{M}(j\omega)\right|\right)^{2} \tag{5}$$

where N is the number of experimental data points considered and $A\hat{M}(\omega)$ is the apparent mass modulus of the model for a given set of parameter values. Mean squared error was chosen because it provides a global energy averaged measure of goodness of fit and because its minimisation ensures the orthogonality of the prediction error (Allen 1971). The m.s.e was calculated using a total of N=56 frequency lines covering the interval from 2 to 45 Hz.

Given a measure of modelling error, a variety of optimisation routines exist for locating minima. Unfortunately, many of the most effective perform global unconstrained optimisation. For mass-spring-damper systems where the parameters have a physical interpretation this can lead to unrealistic or unwanted solutions. In such cases the unconstrained optimisation problem can be transformed into a constrained optimisation by means of a penalty function (Rao 1996) which increases the error measure whenever parameter values exceed predetermined limits. For the child models a first physical restriction that was implemented was that all parameter values should be positive. This was achieved by means of a penalty function which added a large fixed increase in error whenever a negative parameter value was attempted.

$$penalty_1(m_0, m_1, c_1, k_1) = \begin{cases} 10{,}000 & for \;\; m_0 < 0 \\ & \qquad m_1 < 0 \\ & \qquad c_1 < 0 \\ & \qquad k_1 < 0 \\ 0 & otherwise \end{cases} \tag{6}$$

A second restriction was that the individual component masses should sum to approximately the total mass of the child. The penalty function introduced in this case added a large fixed increase in error whenever the sum of the component masses m_0 and m_1 was less than 50% or more than 150% the total mass of the child in question.

$$penalty_2(m_1, m_2) = \begin{cases} 10{,}000 & for \;\; (m_0 + m_1) < 0.5\; m_{child} \\ 0 & for \;\; 0.5\, m_{child} < (m_0 + m_1) < 1.5\, m_{child} \\ 10{,}000 & for \;\; 1.5\; m_{child} < (m_0 + m_1) \end{cases} \tag{7}$$

Use of the penalty functions lead to a total fitness function defined by the sum of the mean squared error and the possible penalties.

$$fitness(m_0, m_1, c_1, k_1) = m.s.e.(m_0, m_1, c_1, k_1) + penalty_1(m_0, m_1, c_1, k_1) + penalty_2(m_0, m_1) \quad (8)$$

The optimisation method chosen was the Differential Evolution (DE) algorithm, a parallel direct search method which operates over continuous parameter spaces (Price and Storn 1997, Storn and Price 1997). Differential Evolution is conceptually similar to Genetic Algorithm optimisation with the principal difference being that all operations are performed using floating point numbers rather than binary strings of zeros and ones. The motivation for choosing DE was its simplicity (it has only three control parameters: *NP*, *F* and *CR*) and its efficiency at identifying both linear and nonlinear systems (Kyprianou et. al. 2000).

DE operates on a population of *NP* vectors held in a primary array, each containing a set of *D* parameter values of the problem being optimised (m_0, m_1, c_1 and k_1 in the case of the single degree of freedom child model). Based on empirical testing, Price and Storn suggest a value for *NP* of 5 to 10 times the number of parameters in the vectors being optimised. In the first generation the values of each vector are assigned a random number, guaranteeing that the initial population spans the parameter space. For each vector of the primary array DE then performs a process which leads to a final comparison between a target vector ($\boldsymbol{P}_{target}$) and a competing trial vector ($\boldsymbol{P}_{trial}$), the fitness functions of the two determining which survives to take the place of the target vector in the successive generation held in a secondary array. By means of successive fitness-based selections, and swaps from the secondary array to the primary array, DE evolves optimal parameter vectors.

Differential Evolution is distinguished from other direct search optimisation procedures by the biologically inspired process which produces the trial vector. A parent vector ($\boldsymbol{P}_{parent}$) from the population of the primary array is mutated by adding noise to its parameters, thus helping to explore new areas of parameter space and to escape from local minima. The noise is taken to be the scaled difference between two other vectors ($\boldsymbol{P}_1$ and $\boldsymbol{P}_2$) chosen randomly from the population of the primary array.

$$P_{mutated} = P_{parent} + F(P_1 - P_2) \quad (9)$$

where *F* is a scaling factor which must be in the range $0 \le F \le 1.2$ for stability and whose optimal value for most problems lies in the range $0.4 \le F \le 1.0$. The vector produced by mutation and the original target vector are then used in a crossover operation designed to resemble the process by which a child inherits DNA from its two parents. Parameter values are exchanged by means of *D-1* binomial experiments which are controlled by the crossover ratio CR, which can have a value $0 \le CR \le 1$. For each parameter, a uniformly distributed random number is generated within the interval from 0 to 1 and the child vector receives a parameter from the original target vector ($\boldsymbol{P}_{target}$) if the number is greater than CR, whereas it receives the parameter from the mutated parent otherwise. By using *D-1* binomial experiments DE ensures that at least one parameter is always taken from the noisy mutated vector. The fitness of the target and trial vectors are then compared and the best survives to pass to the next generation held in the secondary array. Operations for a single generation continue until all vectors of the primary array have been targeted and their corresponding positions in the secondary array filled. By then exchanging the elements of the secondary array for those of the primary array the process can be repeated any number of times. The process is halted when either a maximum number of iterations has been completed or a criteria such as an average fitness value for the population of the secondary array has been achieved. The DE operational sequence is illustrated in Figure 4.

[Insert Figure 4 Here]

4. Results

More than 50 trial runs were performed for each child mass-spring-damper model, leading to the choice of DE algorithm parameters *NP*=70, *F*=0.5 and *CR*=0.5 and 200 fixed iterations, which provided the best results across the data ensemble. Optimisation runs were performed for all children for the input amplitude of 1800 mV corresponding to an r.m.s. input acceleration of 1.2 m/s^2. Only the experimental data from the 1800mV tests was used in the current investigation due to the similarity of the data sets obtained at the two amplitudes and due to the slightly higher signal-to-noise ratio of the 1800 mV tests. The identified parameter values are presented along with the group average values and the mean squared error in Table 2. Figure 5 provides a comparison between the experimentally acquired and the numerically predicted data.

[Insert Table 2 Here] [Insert Figure 5 Here]

It is instructive to compare the mean model parameters of the group of eight small children to the parameters of similar models reported in the literature for primates and adult humans. For a group of four Rhesus monkeys, Smith (1992) reported a mean moving mass m_1 of 6.67 kg, a mean stiffness of 15190 N/m and a mean damping coefficient of 231 Ns/m. For the same parameters, Slonim (1985) reported average values of 15.5 kg, 22400 N/m and 550.1 Ns/m for a group of three Baboons. For a group of 7 to 14 year old children, Wei and Griffin (1998) reported a mean moving mass m_1 of 51.2 kg, a mean stiffness of 44130 N/m and a mean damping coefficient of 1485 Ns/m. The parameters of the child models determined in this study are therefore intermediate between the small Rhesus monkeys and the larger humans, being close in value to the results obtained for Baboons.

5. Discussion

Comparison of the model parameters defined for primates, small children and adults suggests possible changes in the mean vibration response as a function of body mass. The dependence of a biological variable *Y* on body mass *M* is often described by means of an allometric scaling law (West, Brown and

Enquist 1997) of the form

$$Y = Y_0 M^b \tag{10}$$

where Y_0 and b are constants. It was once thought that the scaling exponent b reflected geometric constraints on living organisms and that, as such, the values should be multiples of 1/3. More recently it has been shown that many biological phenomena can be described by scaling exponents which are 1/4 powers of the mass of the organism. Examples include the metabolic rates of living organisms which scale with the +3/4 power of the mass, heartbeat which scales with the –1/4 power and blood circulation and life span which scale with the +1/4 power. West, Brown and Enquist have developed a general theory of 1/4 power scaling for biological organisms which is based on the transport of nutrients through fractal space-filling networks with branching tubes of finite size.

In order to compare the properties of the whole-body vibration models defined for different subjects, Table 3 presents the mechanical parameters, natural frequency and damping ratio of the single degree of freedom with base support mass-spring-damper models reported for primates by Broderson and Von Gierke (1971), by Slonim (1985) and by Smith (1992), reported here for small children, and reported for large children and adults by Wei and Griffin (1998). With masses ranging from 5.98 to 108.0 kg the data set covers more than one order of magnitude in the independent variable. A possible limitation of the proposed comparison is represented by the differences in the original experimental conditions which generated the data which lead to the vibration models. Parameters which vary across the set of studies considered include the test apparatus, the test vibration type, the test vibration amplitude, the sitting posture used (with or without backrest support, seat inclination, etc.) and the muscle tension adopted. Despite the variations in the original test conditions, it was nevertheless thought instructive to compare the models. Partial justification for this position includes the consideration that the effect of several of the test parameters which are known to effect the vibration response of the seated body is smaller than the differences between the various subject types whose models have been gathered together for comparison in the current study.

[Insert Table 3 Here] [Insert Figure 6 Here] [Insert Figure 7 Here]

A first question of interest is whether, for the group of models considered, the moving mass remained proportional to the total mass of the subject. Figure 6 presents the moving mass m_1 plotted against the total model mass m_{total} using logarithmic scales for both axis. Regression produces a scaling exponent of approximately unity, suggesting that the moving mass remained approximately proportional to the total mass across the set of whole-body vibration models. A second question of interest is the growth in the body spring stiffness with mass. Figure 7 presents the single degree of freedom spring constants plotted against body mass. In this case regression suggests that body stiffness grew with a power exponent of approximately +1/2. Care must be taken when interpreting this relationship, however, since the total variance accounted for is 68%.

Of relevance to both the maximum response amplitude and the absorbed power of a single degree of freedom model is the damping level. Figure 8 presents the damping coefficients while Figure 9 presents the damping ratios of the models considered. Regression suggests that, for the models considered, the damping coefficient grew as a function of body mass according to a scaling exponent of approximately +3/4. The dimensionless damping ratio was, instead, found to be independent of body mass, as suggested by values of approximately zero for both the growth exponent and the coefficient of determination. For the models considered, this suggests nearly constant free vibration decay rates and forced vibration magnification factors.

[Insert Figure 8 Here] **[Insert Figure 9 Here]** **[Insert Figure 10 Here]**

Figure 10 presents the natural frequencies determined using the mass and stiffness values of the models, plotted as a function of the total model mass. In this case the relationship is negative, and regression produces an exponent of –0.29, nearly a quarter power exponent of –1/4.

Figures 6 to 10 and the regression analysis results suggest sizeable differences in some parameters, but only negligible differences in others, across the data set of single degree of freedom with base support models considered here. Being summary models determined from the data of physical experiments performed under a variety of test conditions, the current comparisons cannot be interpreted as definitive proof of the underlying allometric relationships. Nevertheless, the identified trends in the summary models do suggest interesting possibilities for further investigation.

6. Conclusions

A single degree of freedom, linear, mass-spring-damper with base support model was fitted to the vertical driving point apparent mass modulus function obtained for each of eight small children under 18 kg. Differential Evolution, a parallel direct search optimisation method which operates over continuous parameter spaces, was used in conjunction with a mean squared error measure and penalty functions to identify optimal model parameter values. The eight child models were characterised by a mean moving mass m_1 of 8.5 kg, a mean body stiffness k_1 of 21131 N/m and a mean damping coefficient c_1 of 329 Ns/m. Comparison to the parameter values of similar models reported in the literature for Rhesus monkeys, Baboons, large children and adults shows that the values for small children are intermediate between the smaller primates and the larger humans. A regression analysis of the model parameters was performed as a function of subject mass for a data set consisting of the eight child models reported here, twelve similar models reported in the literature for primates, and 60 similar models reported in the literature for large children and adults. The moving mass m_1 of the group of models grew with a power exponent of approximately unity, the body stiffness k_1 grew with a power exponent of approximately +1/2, the damping coefficient c_1 grew with a power exponent of approximately +3/4 and the dimensionless damping ratio was independent of subject mass. The natural frequency of the models grew with a power exponent of approximately –1/4.

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Table 1 Test subject characteristics.

Child	**Age (months)**	**Mass (kg)**	**Height (cm)**	**Sex (M/F)**
al	10.0	9.4	76.0	M
im	15.0	11.0	86.0	M
ja	13.0	11.4	80.0	M
ju	3.0	5.2	50.0	F
le	8.5	9.4	73.0	M
ma	7.0	8.0	66.0	F
mo	14.5	10.2	71.0	F
sa	23.0	12.4	85.0	F
Mean	**11.8**	**9.6**	**73.4**	**-**
Std.	**6.1**	**2.2**	**11.7**	**-**

Table 2 Parameter values of the optimal two-mass single degree of freedom model fit to the modulus function of each child.

Child	**m_{total} (kg)**	**m_o (kg)**	**m_1 (kg)**	**k_1 (N/m)**	**c_1 (Ns/m)**	**m.s.e (kg)**
al	9.2	0.8	8.4	21187	350	0.11
im	10.8	1.2	9.6	19242	364	0.11
ja	10.5	0.9	9.6	27357	374	0.34
ju	6.3	1.3	5.0	18420	279	0.05
le	8.3	0.6	7.7	19211	298	0.17
ma	7.6	0.8	6.8	16618	232	0.23
mo	11.8	1.0	10.8	21162	431	0.33
sa	11.2	0.8	10.4	25853	305	0.29
Mean	**9.5**	**0.9**	**8.5**	**21131**	**329**	**0.20**
Std.	**1.9**	**0.2**	**2.0**	**3704**	**63**	**0.11**

Table 3 Parameter values reported for single degree of freedom mass-spring-damper models of Rhesus monkeys, Baboons, small children, large children and adult humans.

Test Subject	m_{total} (kg)	m_o (kg)	m_1 (kg)	k_1 (N/m)	c_1 (Ns/m)	Fn [Hz]	ς
Rhesus (Broderson 1971)	6.57	3.29	3.29	9054	150.6	8.35	0.44
Rhesus 1 (Smith 1992)	7.14	3.08	4.07	16530	249.5	10.15	0.48
Rhesus 2 (Smith 1992)	5.98	2.95	3.04	11340	180.9	9.73	0.49
Rhesus 3 (Smith 1992)	6.70	3.20	3.50	11960	189.5	9.31	0.46
Rhesus 4 (Smith 1992)	6.87	2.36	4.51	19900	306.8	10.57	0.51
Rhesus 058 (Slonim 1985)	6.78	0.83	5.95	29400	404.2	11.19	0.48
Rhesus 314 (Slonim 1985)	6.58	0.81	5.77	41475	439.2	13.49	0.45
Rhesus A360 (Slonim 1985)	7.58	0.93	6.65	15750	311.5	7.75	0.48
Rhesus 318 (Slonim 1985)	13.36	1.64	11.72	14525	642.2	5.60	0.78
Baboon F96 (Slonim 1985)	13.36	1.64	11.72	23975	707.0	7.20	0.67
Baboon F26 (Slonim 1985)	20.94	2.57	18.37	19775	385.0	5.22	0.32
Baboon F88 (Slonim 1985)	15.16	1.86	13.30	23450	558.2	6.68	0.50
al (Giacomin 2002)	9.20	0.80	8.40	21187	350.0	7.99	0.41
im (Giacomin 2002)	10.80	1.20	9.60	19242	364.0	7.13	0.42
ja (Giacomin 2002)	10.50	0.90	9.60	27357	374.0	8.50	0.36
ju (Giacomin 2002)	6.30	1.30	5.00	18420	279.0	9.66	0.46
le (Giacomin 2002)	8.30	0.60	7.70	19211	298.0	7.95	0.39
ma (Giacomin 2002)	7.60	0.80	6.80	16618	232.0	7.87	0.35
mo (Giacomin 2002)	11.80	1.00	10.80	21162	431.0	7.05	0.45
sa (Giacomin 2002)	11.20	0.80	10.40	25853	305.0	7.94	0.29
S1 (Wei and Griffin 1998)	45.90	1.30	44.60	34142	1187.0	4.40	0.48
S2 (Wei and Griffin 1998)	44.40	8.80	35.60	41151	1122.0	5.41	0.46
S3 (Wei and Griffin 1998)	108.00	21.30	86.70	71772	1845.0	4.58	0.37
S4 (Wei and Griffin 1998)	57.10	4.30	52.80	62976	1631.0	5.50	0.45
S5 (Wei and Griffin 1998)	52.80	5.00	47.80	34653	1312.0	4.29	0.51
S6 (Wei and Griffin 1998)	43.90	12.90	31.00	29409	675.0	4.90	0.35
S7 (Wei and Griffin 1998)	72.20	11.70	60.50	54623	1658.0	4.78	0.46
S8 (Wei and Griffin 1998)	52.10	13.00	39.10	35756	1009.0	4.81	0.43
S9 (Wei and Griffin 1998)	48.00	15.30	32.70	36286	898.0	5.30	0.41
S10 (Wei and Griffin 1998)	65.90	0.10	65.80	66748	1705.0	5.07	0.41
S11 (Wei and Griffin 1998)	41.40	5.90	35.60	38962	985.0	5.27	0.42
S12 (Wei and Griffin 1998)	56.20	17.20	39.00	34822	954.0	4.76	0.41
S13 (Wei and Griffin 1998)	65.80	6.80	59.00	70926	1447.0	5.52	0.35
S14 (Wei and Griffin 1998)	60.80	1.40	59.40	54085	1475.0	4.80	0.41
S15 (Wei and Griffin 1998)	55.60	20.90	34.70	71813	1173.0	7.24	0.37
S16 (Wei and Griffin 1998)	53.40	2.10	51.30	46384	1797.0	4.79	0.58
S17(Wei and Griffin 1998)	56.20	4.70	51.50	66593	1377.0	5.72	0.37
S18 (Wei and Griffin 1998)	83.80	3.40	80.40	66803	1833.0	4.59	0.40
S19 (Wei and Griffin 1998)	58.90	12.20	46.70	42940	1286.0	4.83	0.45
S20 (Wei and Griffin 1998)	78.30	2.10	76.20	77829	2345.0	5.09	0.48
S21 (Wei and Griffin 1998)	60.10	13.60	46.50	48025	1165.0	5.11	0.39
S22 (Wei and Griffin 1998)	47.00	3.10	43.90	42443	1083.0	4.95	0.40
S23 (Wei and Griffin 1998)	58.00	17.30	40.70	52609	1204.0	5.72	0.41
S24 (Wei and Griffin 1998)	44.20	0.90	43.30	63948	1636.0	6.12	0.49
S25 (Wei and Griffin 1998)	43.70	4.80	38.90	26951	957.0	4.19	0.47
S26 (Wei and Griffin 1998)	59.80	11.70	48.10	48045	1217.0	5.03	0.40
S27 (Wei and Griffin 1998)	45.80	0.50	45.30	58890	1486.0	5.74	0.45

S28 (Wei and Griffin 1998)	51.70	3.10	48.60	40143	1565.0	4.57	0.56
S29 (Wei and Griffin 1998)	41.20	1.50	39.70	58186	1277.0	6.09	0.42
S30 (Wei and Griffin 1998)	53.20	0.40	52.80	37755	1792.0	4.26	0.63
S31 (Wei and Griffin 1998)	52.00	12.10	39.90	36342	1170.0	4.80	0.49
S32 (Wei and Griffin 1998)	53.30	1.10	52.20	38886	1925.0	4.34	0.68
S33 (Wei and Griffin 1998)	51.70	18.20	33.50	32252	621.0	4.94	0.30
S34 (Wei and Griffin 1998)	50.30	13.80	36.50	32174	935.0	4.73	0.43
S35 (Wei and Griffin 1998)	52.90	2.20	50.70	45515	1403.0	4.77	0.46
S36 (Wei and Griffin 1998)	56.60	15.40	41.20	38227	1178.0	4.85	0.47
S37 (Wei and Griffin 1998)	51.20	16.70	34.50	43578	976.0	5.66	0.40
S38 (Wei and Griffin 1998)	47.70	8.20	39.50	35351	1076.0	4.76	0.46
S39 (Wei and Griffin 1998)	55.80	2.60	53.20	46037	1577.0	4.68	0.50
S40 (Wei and Griffin 1998)	40.70	11.90	28.80	39493	823.0	5.89	0.39
S41 (Wei and Griffin 1998)	48.00	9.60	38.40	50712	1172.0	5.78	0.42
S42 (Wei and Griffin 1998)	64.50	17.40	47.10	30671	880.0	4.06	0.37
S43 (Wei and Griffin 1998)	60.30	1.60	58.70	52524	1319.0	4.76	0.38
S44 (Wei and Griffin 1998)	43.90	12.10	31.80	52151	1003.0	6.45	0.39
S45 (Wei and Griffin 1998)	52.30	14.90	37.40	35154	826.0	4.88	0.36
S46 (Wei and Griffin 1998)	50.30	0.10	50.20	38850	1435.0	4.43	0.51
S47 (Wei and Griffin 1998)	43.70	0.80	42.90	39338	1909.0	4.82	0.73
S48 (Wei and Griffin 1998)	59.60	18.80	40.80	42586	994.0	5.14	0.38
S49 (Wei and Griffin 1998)	31.90	12.50	19.40	34387	421.0	6.70	0.26
S50 (Wei and Griffin 1998)	33.40	7.00	26.40	32487	762.0	5.58	0.41
S51 (Wei and Griffin 1998)	23.40	2.10	21.30	24887	511.0	5.44	0.35
S52 (Wei and Griffin 1998)	33.30	2.20	31.10	37977	923.0	5.56	0.42
S53 (Wei and Griffin 1998)	34.40	3.80	30.60	36203	992.0	5.47	0.47
S54 (Wei and Griffin 1998)	30.20	5.50	24.70	28960	734.0	5.45	0.43
S55 (Wei and Griffin 1998)	31.30	3.10	28.20	35428	753.0	5.64	0.38
S56 (Wei and Griffin 1998)	51.20	0.30	50.90	47668	1564.0	4.87	0.50
S57 (Wei and Griffin 1998)	46.00	11.10	34.90	25937	820.0	4.34	0.43
S58 (Wei and Griffin 1998)	45.10	14.20	30.90	31973	607.0	5.12	0.31
S59 (Wei and Griffin 1998)	31.00	3.50	27.50	33395	718.0	5.55	0.37

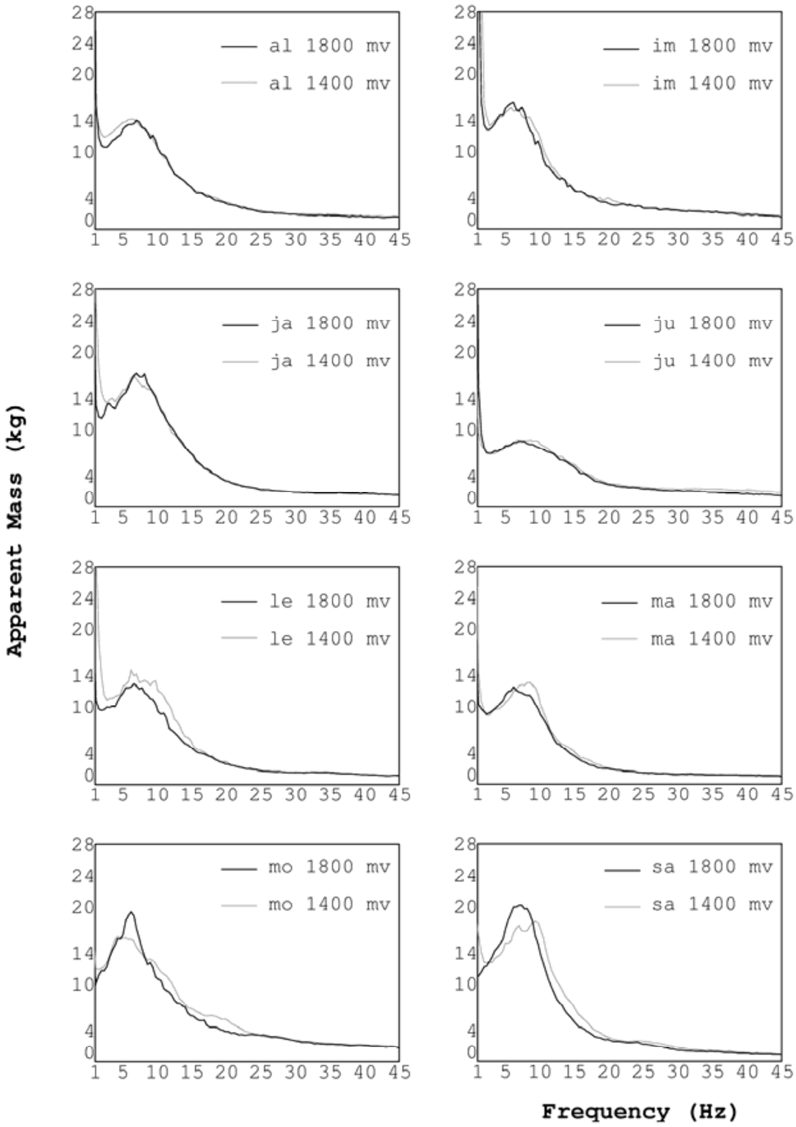


Figure 1 Apparent mass modulus functions for eight children tested at r.m.s. acceleration levels of 0.8 m/s^2 (1400mV) and 1.2 m/s^2 (1800 mV).

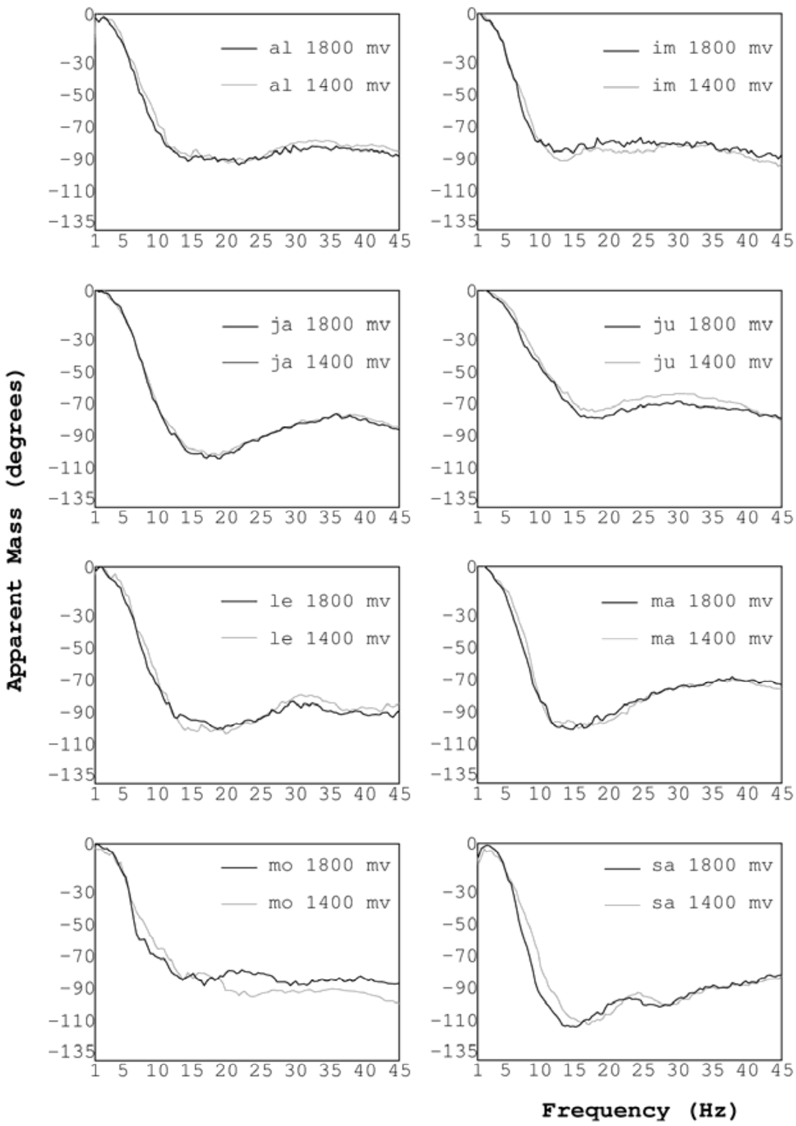


Figure 2 Apparent mass phase functions for eight children tested at r.m.s. acceleration levels of 0.8 m/s^2 (1400mV) and 1.2 m/s^2 (1800 mV).

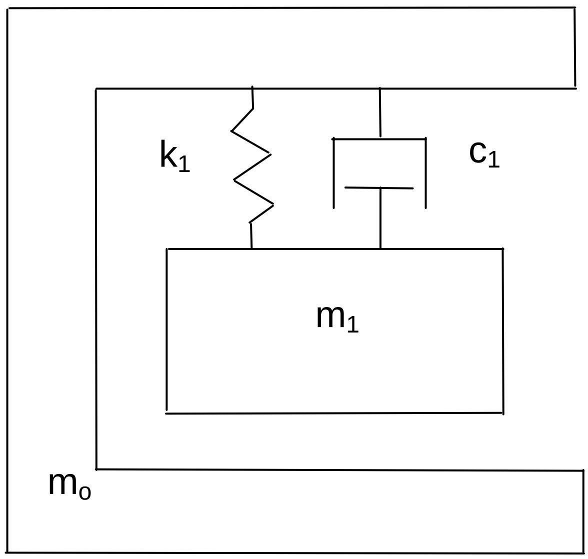
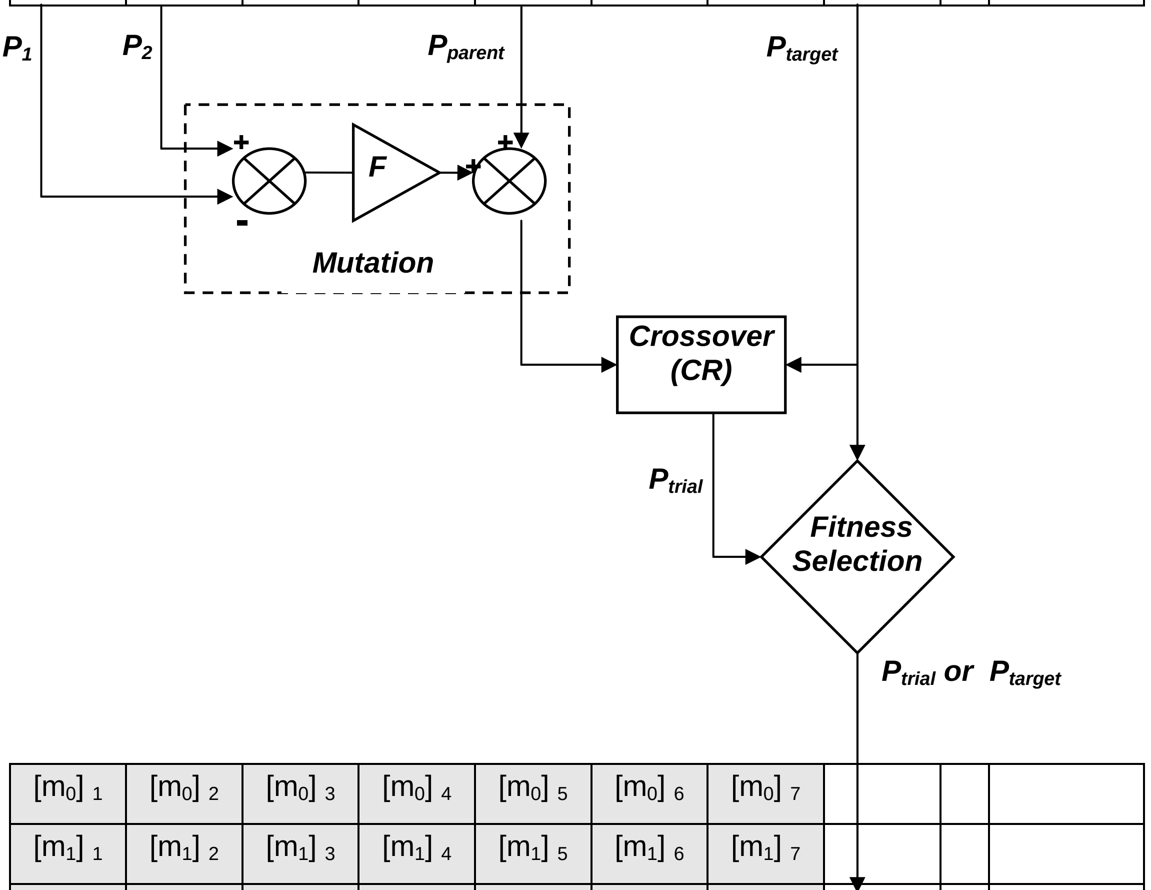


Figure 3 Single degree of freedom with base support mass-spring-damper model

First generation of *NP* parameter vectors with associated fitness values (primary array).

$[m_0]_1$	$[m_0]_2$	$[m_0]_3$	$[m_0]_4$	$[m_0]_5$	$[m_0]_6$	$[m_0]_7$	$[m_0]_8$	…	$[m_0]_{NP}$
$[m_1]_1$	$[m_1]_2$	$[m_1]_3$	$[m_1]_4$	$[m_1]_5$	$[m_1]_6$	$[m_1]_7$	$[m_1]_8$	…	$[m_1]_{NP}$
$[c_1]_1$	$[c_1]_2$	$[c_1]_3$	$[c_1]_4$	$[c_1]_5$	$[c_1]_6$	$[c_1]_7$	$[c_1]_8$	…	$[c_1]_{NP}$
$[k_1]_1$	$[k_1]_2$	$[k_1]_3$	$[k_1]_4$	$[k_1]_5$	$[k_1]_6$	$[k_1]_7$	$[k_1]_8$	…	$[k_1]_{NP}$
fitness $_1$	fitness $_2$	fitness $_3$	fitness $_4$	fitness $_5$	fitness $_6$	fitness $_7$	fitness $_8$	…	fitness $_{NP}$



$[m_0]_1$	$[m_0]_2$	$[m_0]_3$	$[m_0]_4$	$[m_0]_5$	$[m_0]_6$	$[m_0]_7$			
$[m_1]_1$	$[m_1]_2$	$[m_1]_3$	$[m_1]_4$	$[m_1]_5$	$[m_1]_6$	$[m_1]_7$			
$[c_1]_1$	$[c_1]_2$	$[c_1]_3$	$[c_1]_4$	$[c_1]_5$	$[c_1]_6$	$[c_1]_7$			
$[k_1]_1$	$[k_1]_2$	$[k_1]_3$	$[k_1]_4$	$[k_1]_5$	$[k_1]_6$	$[k_1]_7$			
fitness $_1$	fitness $_2$	fitness $_3$	fitness $_4$	fitness $_5$	fitness $_6$	fitness $_7$			

Next generation of *NP* parameter vectors with associated fitness values (secondary array).

Figure 4 One trial of the DE algorithm as applied to the child model parameter estimation task.

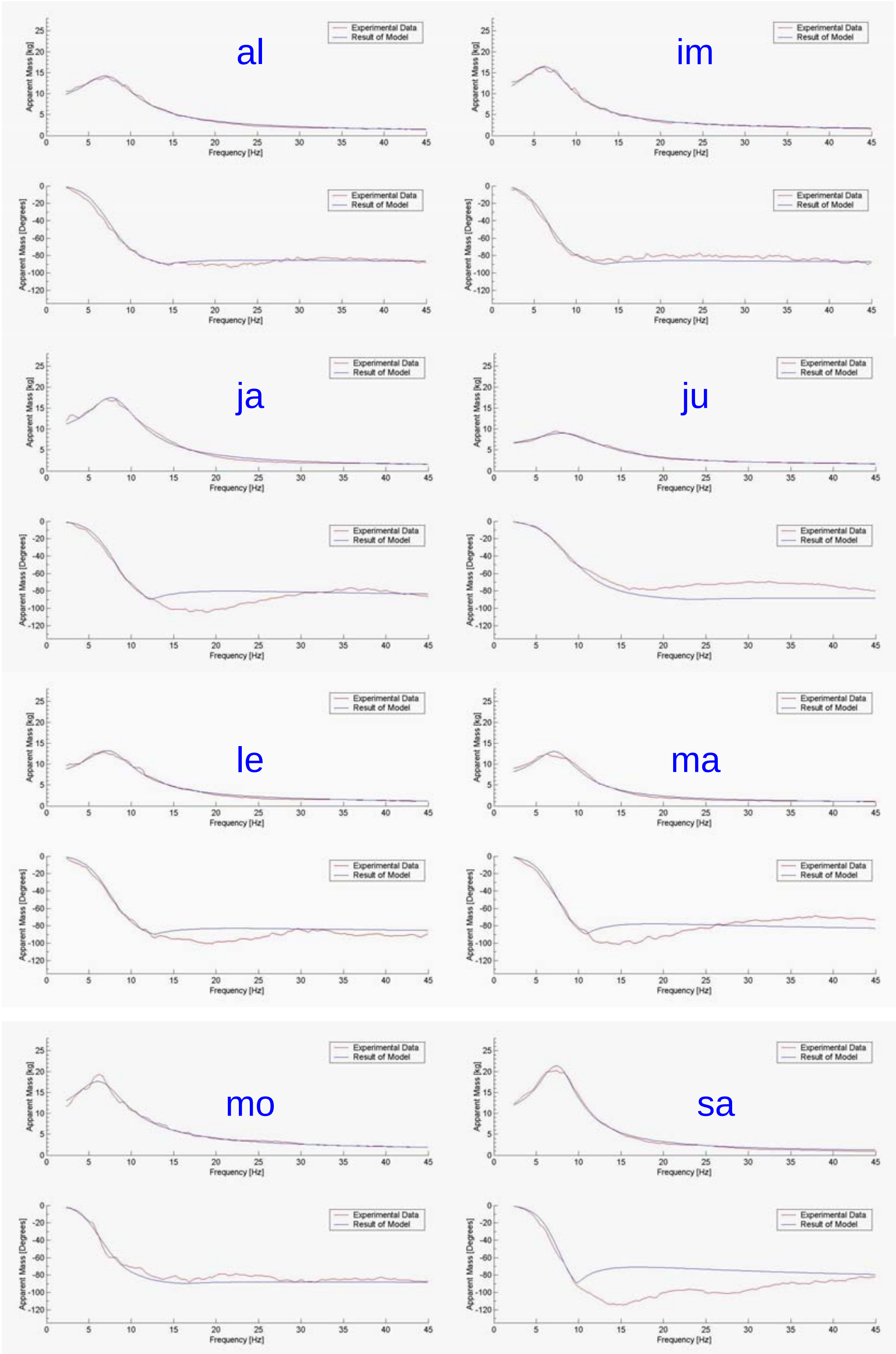


Figure 5 Single degree of freedom modelling results for eight small children.

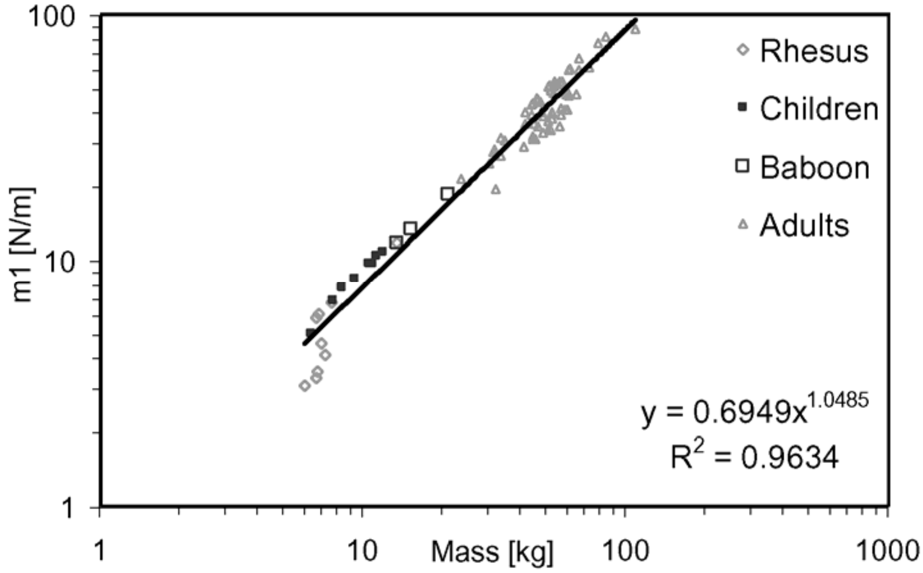


Figure 6 Moving mass m_1 as a function of the total mass for single degree of freedom models of primates, small children and adult humans.

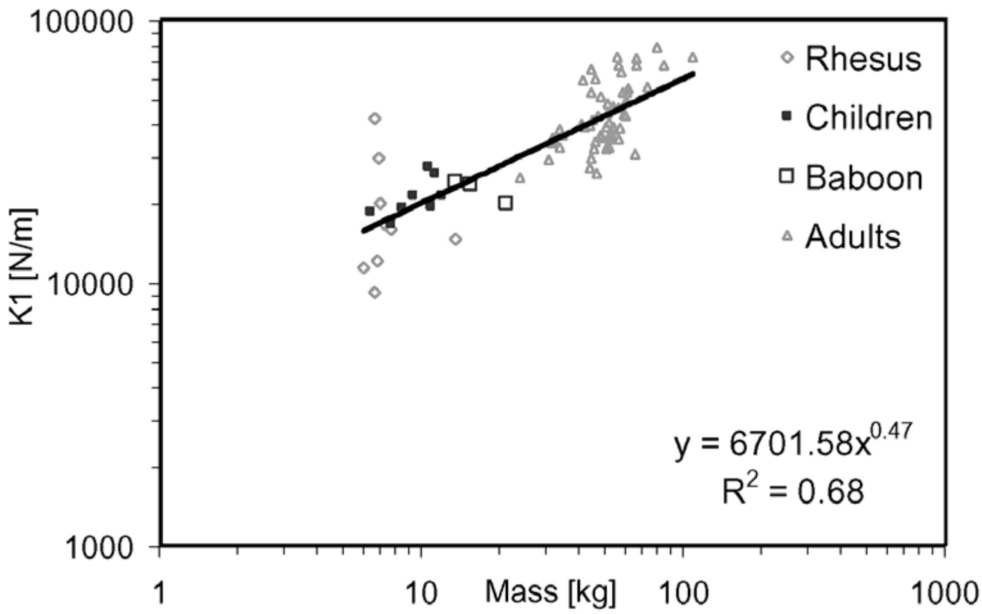


Figure 7 Body stiffness k_1 as a function of the total mass for the single degree of freedom models of primates, small children and adult humans.

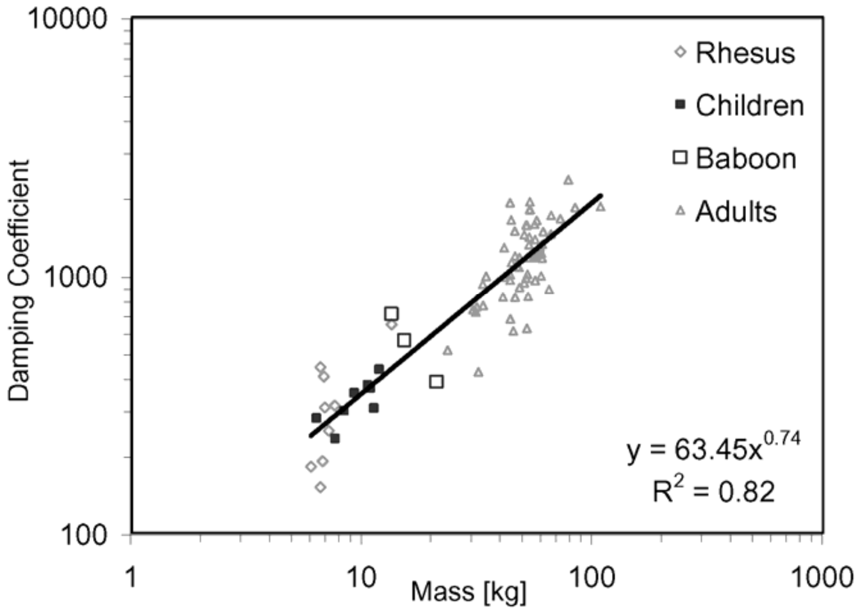


Figure 8 Damping coefficient c_1 as a function of the total mass for the single degree of freedom models of primates, small children and adult humans.

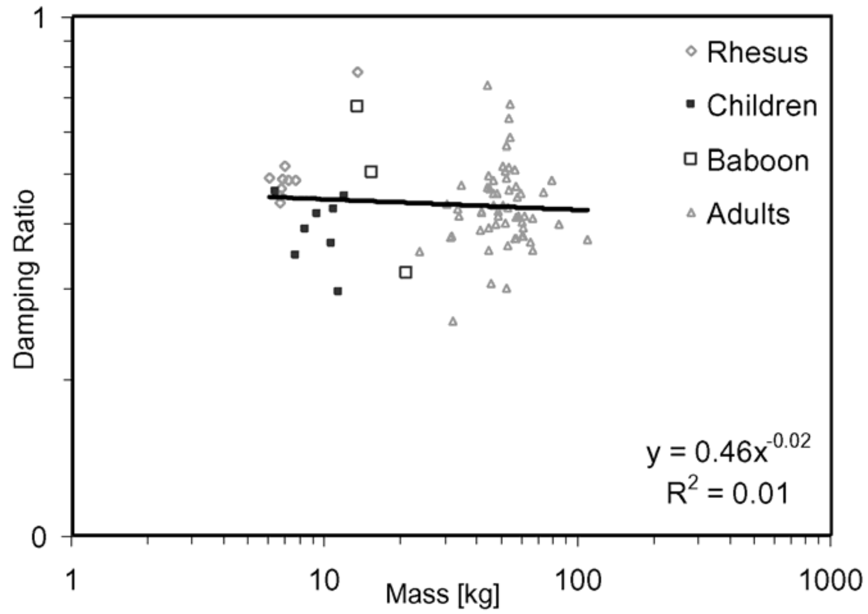


Figure 9 Damping ratio as a function of the total mass for the single degree of freedom models of primates, small children and adult humans.

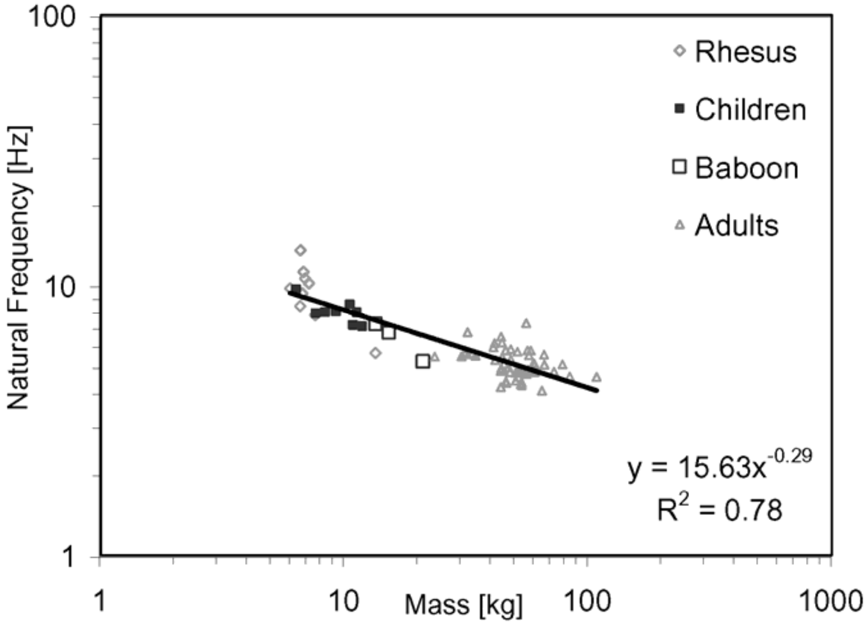


Figure 10 Natural frequency as a function of the total mass for the single degree of freedom models of primates, small children and adult humans.