APPLICATION OF THE WAVELET BASED FRFs TO THE ANALYSIS OF NONSTATIONARY VEHICLE DATA

W.J. Staszewski and J. Giacomin
Dynamics Research Group
Department of Mechanical Engineering
University of Sheffield
Mappin Street
Sheffield S1 3JD, United Kingdom
E-mail: w.j.staszewski@sheffield.ac.uk
E-mail: j.a.giacomin@sheffield.ac.uk

ABSTRACT. *A traditional difficulty in vehicle road testing is the nonstationary nature of the data. Frequency Response Functions (FRF) and other classical input/output relationships are difficult to obtain accurately due to the uncontrolled time and energy characteristics of the road input to the vehicle. Time-variant methods can be useful in this analysis. This paper presents a new FRF approach based on the wavelet transform. The procedure employs the continuos Grossman-Morlet wavelets calculated in the frequency domain. The method is used to analyse acceleration transmissibilities across an automobile seat in the vertical direction. The study involves transient road data from a speedbump and nonstationary road data from a pave' surface. The method provides new insights with respect to classical time-invariant approaches and would thus seem promising for a wide range of vehicle testing applications. Several observations are made regarding the dynamics of the person/seat system.*

NOMENCLATURE

a	: dilation
b	: translation
C_ψ	: admissibility coefficient
f	: frequency
f_0	: central frequency
i	: $\sqrt{-1}$
t	: time
$x(t)$	: time domain input signal
$y(t)$	: time domain output signal
$H_w(f,t)$	: wavelet based FRF
$W_\psi^x(a,b)$	: wavelet coefficients
$X(f)$	: Fourier transform of $x(t)$
$\psi(t)$	: wavelet function
$\psi_{a,b}(t)$	: scaled wavelet function
$\Psi(f)$	: Fourier transform of $\psi(t)$
Δf	: bandwidth of the wavelet analysis
Δt	: time duration of the wavelet analysis
Δf_ψ	: frequency resolution of the wavelet
Δt_ψ	: time resolution of the wavelet
$\Re$	: set of real numbers
$*$	: complex conjugate

1 INTRODUCTION

A traditional difficulty associated with vehicle road testing and many other forms of field testing is the non-stationary nature of the vibration data [1]. Many vibration tests provide signals whose statistics vary as a function of time. One of the leading makers of vibro-acoustic test systems used by the automotive industry has defined [2] three basic classes of non stationary signal, these classes being evolutionary harmonic signals, evolutionary broadband signals and transient signals. Techniques such as adaptive resampling, kalman filtering or prony techniques are recommended for the analysis of evolutionary harmonic signals. Autoregressive signal modelling methods are suggested in the case of evolutionary broadband signals, while the wavelet transform and Wigner-Ville distribution are suggested for the analysis of transient signals. While several of these methods are widely used in industry, few published papers demonstrate the usefulness of wavelet techniques for the analysis of vehicle road data. Specifically, the authors know of none to date which analyse data relative to automobile seats.

This paper describes the application of wavelets to the analysis of road data relative to automobile seats. The objective of this study was to verify the usefulness of time-variant techniques for the analysis of vehicular road data, and to better understand the time-variant properties of the person/seat system. The *FRF* based on the continuous Grossman-Morlet wavelets is introduced in Section 2. For the sake of completeness the wavelet analysis is presented in Section 3. Section 4 describes the experimental analysis. This involves the data from road tests over a speedbump which exemplifies a typical roadway transient and a a pave' (cobblestone) surface which provided continuous, but nonstationary vibration characteristics. Sections 5 and 6 present a discussion of the obtained results and the conclusions.

2 WAVELET BASED *FRF*

The input-output relation of a system can be represented using the *FRF*. This well known and established representation holds for linear systems and requires an extension for nonlinear and/or nonstationary processes. Nonstationary processes can be analysed in time-frequency or time-scale domains. This approach allows one to extend classical impulse and frequency response functions to two dimensional analysis. The generalised *FRF* based on the evolutionary spectrum was proposed by Priestley [3]. It is defined in the time-frequency domain and expresses in general variations of the system response as time evolves. Similar representation can be proposed when the time-scale domain is used.
The wavelet based *FRF* can be defined as

$$H_w(f,t) \;=\; \frac{W_\psi^y(a,b)}{W_\psi^x(a,b)} \tag{1}$$

where $W_\psi^x(a,b)$ and $W_\psi^y(a,b)$ are the wavelet transforms of the input and the output of the system respectively. This equation can be modified in practice as

$$H_w(f,t) \;=\; \frac{W_\psi^y(a,b)\;\; W_\psi^{x^*}(a,b)}{W_\psi^x(a,b)\;\; W_\psi^{x^*}(a,b)} \tag{2}$$

where $*$ denotes the complex conjugate of the function. The calculations of the wavelet based *FRF* leads to the wavelet analysis described in the next section.
The wavelet based FRF represents the ratio of output-to-input in the time-scale domain and thus fully characterises time-variant physical systems. It is is understood that all operations involving the Impulse Response Function (*IRF*) and the *FRF* which hold for one dimension can be expanded to create the necessary framework for two dimensions.
In the experimental analysis presented below this input-output relationship is used in terms of the transmissibility. Since the square of the modulus of the wavelet transform can be interpreted as an energy density distribution over the (a,b) time-scale plane, the wavelet based transmissibility gives the flow of the energy in the time-scale (time-frequency) domain. This flow is studied in the paper using the concept of the ridges of $H_w(f,t)$. Mathematically the ridge of the function is the curve which consists of some stationary points of the function related directly to the function maxima. For linear systems the ridge of the function is the distribution of its local maxima. The theory of this concept is beyond the scope of the paper, more details can be found in [4]. The wavelet based *FRF* is used in the experimental analysis presented in Section 4.

3 WAVELET ANALYSIS

For the sake of completeness wavelet analysis is introduced in this section. For more details the reader is referred to [5]. Wavelets are mathematical functions $\psi(t)$ which are used to decompose a signal $x(t)$ into scaled wavelet coefficients $W_\psi^x(a,b)$. This decomposition can be expressed mathematically as

$$W_\psi^x(a,b) \;=\; \int_{-\infty}^{+\infty} x(t)\; \psi_{a,b}^*(t)\; dt \tag{3}$$

where $\psi_{a,b}^*(t)$ are the scaled wavelets and $\psi^*(\cdot)$ is the complex conjugate of $\psi(\cdot)$ and $\psi_{a,b}^*(t)$ The family of scaled wavelet functions can be generated from the wavelet function by shifts in the time domain (b - translation) and scaling in the scale domain (a - dilation)

$$\psi_{a,b}(t) \;=\; \frac{1}{\sqrt{a}}\,\psi\left(\frac{t-b}{a}\right), \qquad a>0,\; b\in\Re \tag{4}$$

For the function $\psi(t)$ to qualify as a wavelet, it must satisfy the admissibility condition given by

$$0 \;<\; C_\psi \;=\; \int_{-\infty}^{+\infty} \frac{|\Psi(f)|^2}{|f|}\, df \;<\; \infty \tag{5}$$

where $\Psi(f)$ is the Fourier transform of $\psi(t)$. The locality of the transform requires that $\psi(t)$ decays at infinity

$$\int_{-\infty}^{+\infty} |\psi(t)|\; dt \;<\; \infty \tag{6}$$

Some regularity of the function and its vanishing moments are also required in practice. There are many functions used as wavelets. In this analysis, the Morlet wavelet [5] is used, being defined as

$$\psi(t) \;=\; e^{i2\pi f_o|t|}\; e^{-\frac{|t|^2}{2}} \tag{7}$$

It can be seen that the wavelet transform involves scales rather than frequencies. This analysis cannot be rigorously compared to any time-frequency representation. However there is a relationship between a scale parameter a and frequency f. For the Morlet wavelet function and the same sampling frequencies of the signal and the wavelet, the relationship can be given as [6] $a \;=\; f_0/f$.
The wavelet transform has a number of interesting properties which are widely studied and can be found elsewhere [5]. The wavelet transform, in contrast to some other time-frequency methods (e.g the Wigner-Ville distribution), is a linear representation of a signal which offers time-frequency localisation properties. The compact

support of the wavelet function, given by Equation (6), and the assumed fast decay of $\psi(a,b)$ introduces locality in the time domain. The frequency localisation is clearly seen when the wavelet transform definition is expressed in the frequency domain

$$W_{\psi}^{x}(a,b) = \sqrt{a} \int_{-\infty}^{+\infty} X(f)\ \Psi_{a,b}^{*}(af)\ e^{i2\pi fb}\ df \qquad (8)$$

The local resolution of the wavelet transform in time and frequency is determined by the duration and bandwidth of analysing functions given by [5] $\Delta t = a\,\Delta t_{\psi}$ and $\Delta f = \frac{\Delta f_{\psi}}{a}$, where Δt_{ψ} and Δf_{ψ} are the duration and bandwidth of the basic wavelet function, respectively. It is obvious that the resolution depends on dilation a.

4 EXPERIMENTAL ANALYSIS

The wavelet based *FRF* was used in the experimental study of the vehicle road data. The experiment consisted of a series of test runs over two previously selected road surfaces in the city of Sheffield (**Figure 1**)). The two surfaces selected for the study were a speedbump (Rampton Road) and a pave' surface (Mary Street). The speedbump provided a low frequency transient input to the vehicle while the pave' surface was selected to provide a modest to high frequency nonstationary input to the vehicle. The test speeds were selected so as to be representative of typical vehicle use and to generate significant vibrational input as shown in previous studies [1,7,8].

The experiment consisted of measuring the acceleration data at the rear mounting bolt of the left guide of the driver's seat by means of a PCB model $336C04$ accelerometer and measuring the acceleration at the person/seat interface at the cushion by means of a second PCB model $336C04$ accelerometer mounted to a sit bar [9]. The accelerometers were mounted so as to measure the acceleration in the vertical direction, which was chosen for the experiment since this is the axis which normally attains the highest vibrational levels in automobiles. The accelerometer signals were amplified by means of a PCB model $483A$ amplifier rack then were recorded using *a Kyowa* RTP 610 videocassette data recorder. Both the amplifier rack and the tape recorder were run from an independent $24V$ battery source so as to minimise any noise from the automobile electronic systems.

The test automobile was *a Rover* 214 SLI with $175/SR14$ radial tyres. The vehicle had 114 276 km on the odometer, suspensions were efficient, tyres were also efficient and inflated at the factory recommended pressure. During all tests there were two people in the vehicle, one driving and one running the recorder which was mounted on the rear seat. The driver was female, weighed 62.3 kg and was 1.68 m in height. The passenger running the recorder was male, weighed 85 kg and was 1.80 m in height. The tests consisted of 5 runs over the pave' surface (giving roughly 25 seconds of data for each run) at the fixed speed of 40 km/h and 7 passes over the speedbump at 20 km/h. The speed was controlled by the driver as best possible using the vehicle's instrumentation but some variation was unavoidable, especially during the impact with the speed bump.

A first analysis of the data was performed in the laboratory using the throughput monitor [10] of the *LMS CADA-X* revision 3.4 software system. The *LMS* software was run on an *HP* model 715/64 workstation and a *B&K Type* 7517 front-end was used. The data was sampled from the recorder at 300 Hz and a low pass filter was applied which had a 150 Hz cut-off frequency. The data was then downloaded from *LMS* and analysed on proprietary softwares.

5 RESULTS AND DISCUSSION

An example of the acceleration data for the speedbump test is given in **Figure 2**. The filtering action of the person/seat system can be clearly seen by the reduction of high frequency components in the person/seat interface acceleration signal. **Figures 3a** and **3c** present the wavelet transform of the seat guide acceleration signal (input) and the wavelet transform of the acceleration at the person/seat interface (output), respectively. The wavelet transform ridges for these results can be seen in **Figure 3b** and **3d**, respectively. Here it can be seen that there is time variation present in both the input and output signals with the peak frequency of the input signal initially lowering in frequency then increasing again. The initial lowering of the frequency is probably evidence of the speed drop as the vehicle is slowed by the impact against the obstacle, while the successive rise in frequency is probably evidence of both an increase in vehicle speed as it leaves the bump and a change in the vibration characteristics of the vehicle due to the position of the nonlinear suspension system. Observation of **Figure 3d** shows that the person/seat system appears to introduce an interesting nonlinearity of its own since it can be seen that the change in peak frequency at the person/seat interface does not follow that of the input. A number of sources of seat nonlinearity are known in the literature [8,9] but discussion here would be beyond the scope of the present paper.

The wavelets transform of the analysed input/output data was used to obtain the transmissibility function. The result is presented as the $3D$ plot in **Figure 4a**. To avoid the noise interaction, the values of transmissibility were computed only for the areas where the values of the wavelet transform of the input signal were significant (greater then 0.1 of the maximum). This was done so as to limit analysis to only regions with high signal-to-noise ratio. Two high gain areas at the logarithmic scale of about 0.4 and 0.7 (2.5 Hz and 4.4 Hz) can be observed in the wavelet based transmissibility function. It can be also seen that there is significant variation in the time behaviour of the person/seat system.

The ridges of the transmissibility are presented in **Figure 4b**. These ridges are also limited only to the high levels of the wavelet transform of the input data given in **Figure 3a**. The ridges of the transmissibility function show that the peak transmissibilities are also time functions. The upper ridge at about 0.7 (4.4 Hz) of the logarithmic scale represents the person/seat main resonance. The lower ridge at about 0.4 (2.2 Hz) is an usual feature for a person/seat system and requires further study to identify its causes which may stem from the dynamics in the for-aft direction. An interesting observation is that the person/seat main resonance frequency shows variations of up to 0.8 Hz. This has important implications to seat design as it indicates that attempts to tailor the dynamic response of the seat to improve ride comfort by means of vibration absorbers of other fixed frequency systems might not provide the hoped for attenuation. The results of the current study suggest that the nonlinear nature of person/seat interaction adds another source of variability in the frequency location of the main person/seat resonance. Studies in the literature [9] have shown that this first resonance can be anywhere in the range from $3.5 - 5.0$ Hz depending on the human test subject. The current results show variations of up to 0.8 Hz for the same subject as the vibration level changes. This variation cannot be controlled even if the seat were adjusted for each specific occupant.
Figure 5 presents the wavelet transforms for the pave' data. The high frequency filtering action of the seat (which strongly attenuates at frequencies above 10 Hz) can clearly be seen in **Figure 5b**. A very interesting observation from the data is that the first (logarithmic scale: 0.7, frequency: 4.4 Hz) and the second (logarithmic scale: 1.0, frequency: 8.8 Hz) resonance frequencies of the person/seat system are not continuously excited over the data segment analysed even though fourier analysis shows that much energy is present on average at these frequencies. Some frequency variations of these resonances can also be observed. This indicates that the vibrational environment is changing strongly as a function of time, and that average results from fourier analysis, while useful, should be interpreted in this context. The intermittent nature of the road vibration suggests that techniques such as the ones presented in this paper might provide useful tools also for what regards psychophysical studies in which subjective comfort is correlated with vibrational disturbances.

6 CONCLUSIONS

The wavelet based *FRF* has been introduced and applied to vehicle road data. The experimental analysis involved transient road data from a speedbump and nonstationary road data from a pave' surface. The results give new insights with respect to classical time-invariant approaches. For what regards the dynamics of the person/seat system several useful conclusions can be drawn, the first being that the nonlinear nature of the system leads to important changes (up to 0.8 Hz in the current study) in the first resonance frequency. This is important since it adds another source of variability to the already well documented differences caused by different seat occupants. A second conclusion is that the various person/seat resonances are not continuously excited by road excitation. This may have important implications for psychophysical testing to develop new seat vibrational comfort criteria.
As a final comment the authors would like to suggest that a number of further applications of the methods described in this paper could be useful in the context of vehicle road testing. A number of vibrational problems can occur in road vehicles during transient events such as vehicle acceleration/deceleration, bumps and potholes, gear changes, lane changes and cold start. The methods described in the current paper could be useful towards the analysis of a number of such problems. Planned extensions of the current study include the application of the described methods to person/seat accelerations in the longitudinal and lateral directions and to other road surfaces. Planned developments of the method include the study of techniques for reducing the effects of areas of low signal-to-noise ratio and a study of the functional forms of the ridge functions so as to use these as a means of data reduction and synthesis. This last concept may have possible applications in the field of vibrational mission synthesis.

ACKNOWLEDGEMENTS

The authors would like to thank Mrs. Heather Worden for providing and driving the test vehicle.

REFERENCES

1. Giuliano, F. and Ugo, A., Experimental assessment of ride comfort: a statistical approach, ATA 2nd International Conference on Vehicle Comfort, Bologna (1992)

2. Leuridan, J., Van Derr Auweraer, H. and Vold, H., The analysis of non-stationary dynamic signals, LMS International Application Note (1994)

3. Priestley, M.B., Non-linear and Non-stationary Time series Analysis, Academic Press, London (1988)

4. Staszewski, W.J., Nonlinear Analysis Based on Ridges and Skeletons of the Wavelet Transform, 4^{th} International Congress on Sound and Vibration, St. Petersburg, Russia, 24-27 June (1996)

5. Chui, Ch.K., An Introduction to Wavelets. Wavelets Analysis and its Applications, vol. 1, Academic Press, Boston (1992)

(b)

Figure 1. Two road surfaces used in the current study (a) Speedbump (Rampton Road) (b) Pave' (Mary Street).

6. Staszewski, W.J and Tomlinson, G.R., Application of the Wavelet Transform to Fault Detection in a Spur Gear, Mechanical Systems and Signal Processing, vol. 8(3), 289-307, (1994)

7. Brite-Euram project 5549, 12 month report (1993)

8. Giacomin, J. and Bracco, R., An experimental approach for the vibration optimisation of automotive seats, ATA 3rd Int. Conf. on Vehicle Comfort and Ergonomics, Bologna, Italy, March 29-31 (1995)

9. Griffin, M.J., Handbook of human vibration, Academic Press Publishers, London (1990)

10. LMS CADA-X Time Data Processing Monitor Manual, Revision 3.4, LMS International, Leuvan (1996)

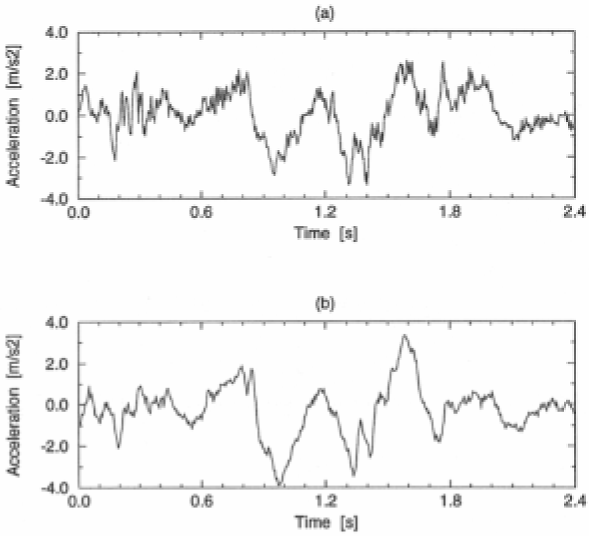

Figure 2. Acceleration data from the speedbump test (a) input at seat guide (b) output at person/seat interface.

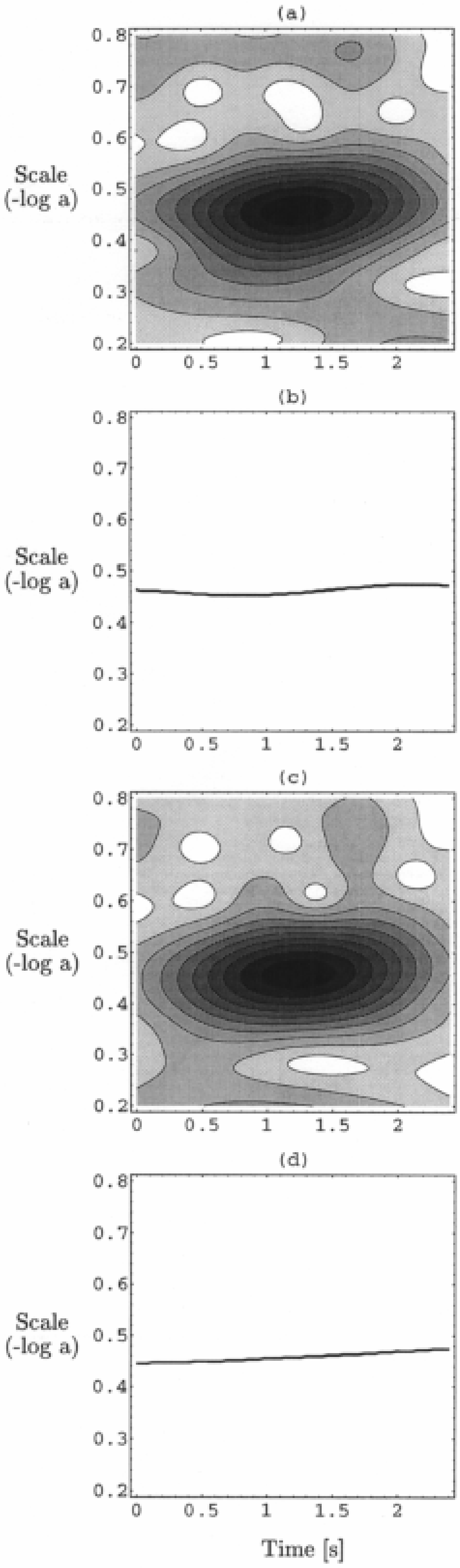

Figure 3. Wavelet results for the speedbump test: (a) wavelet modulus for the input data (b) wavelet ridge for the input data (c)wavelet modulus for the output data (b) wavelet ridge for the output data.

Log scale - frequency conversion:

$0.2 \rightarrow 0.72Hz$, $0.3 \rightarrow 1.75Hz$, $0.4 \rightarrow 2.20Hz$, $0.5 \rightarrow 2.77Hz$, $0.6 \rightarrow 3.48Hz$, $0.7 \rightarrow 4.40Hz$, $0.8 \rightarrow 5.52Hz$.

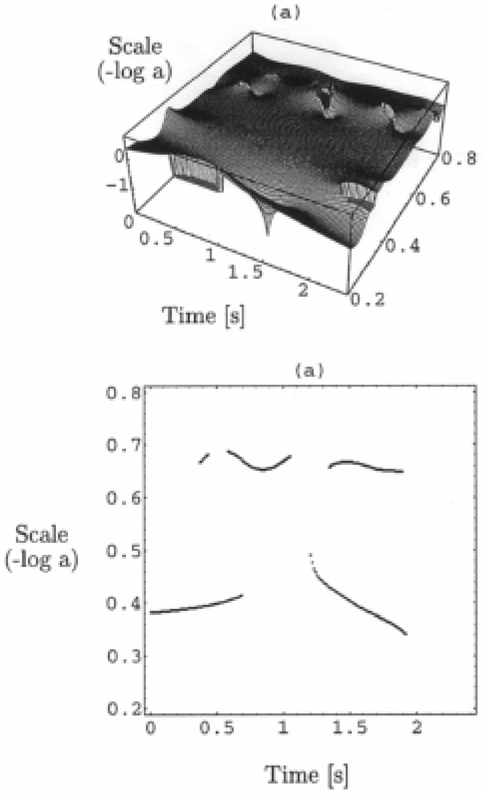

Figure 4. Transmissibility results for the speedbump test: (a) wavelet based transmissibility (b) ridges of the transmissibility.

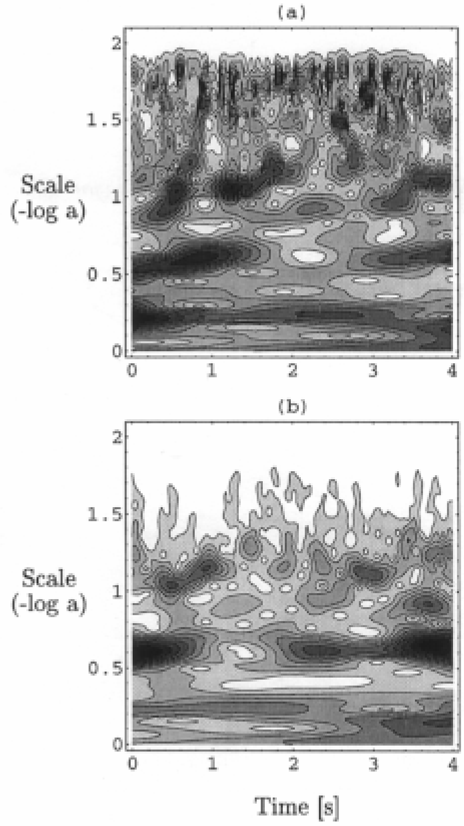

Figure 5. Wavelet results for the pave' test: (a) wavelet modulus for the input data (b)wavelet modulus for the output data.

Log scale - frequency conversion:

$0.2 \rightarrow 0.72Hz$,
$0.3 \rightarrow 1.75Hz$,
$0.4 \rightarrow 2.20Hz$,
$0.5 \rightarrow 2.77Hz$,
$0.6 \rightarrow 3.48Hz$,
$0.7 \rightarrow 4.40Hz$,
$0.8 \rightarrow 5.52Hz$.

Log scale - frequency conversion:

$0.5 \rightarrow 2.77Hz$,
$1.0 \rightarrow 8.75Hz$,
$1.5 \rightarrow 27.67Hz$,
$2.0 \rightarrow 87.5Hz$.